Using Intuitionistic Logic as a Basis for Legal Ontologies

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1. INTRODUCTION

Classical Description Logic has been widely used as a basis for ontology creation and reasoning in many knowledge specific domains. These specific domains naturally include Legal Artificial Intelligence. As in any other domain, consistency is an important issue for legal ontologies. However, due to their inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in other domains. Consistency, or absence of logical contradictions, seems more difficult to maintain when more than one law system can judge a case. This is called a conflict of laws. There are some legal mechanisms to solve these conflicts, some of them stating privileged fori, other ruling jurisdiction, etc. In most of the cases, the conflict is solved by admitting a law hierarchy or a law precedence. Even using these mechanisms, coherence is still a major issue in legal systems. Each layer in this legal hierarchy has to be consistent. Since consistency is a direct consequence of how one deals with logical negation, negation is also a main concern of legal systems.

Negation and subsumption play a central role in ontology coherence. An adequate intuitionistic semantics for negation in a legal domain comes to the fore when we take legally valid individual statements as the inhabitants of our legal ontology. This allows us to elegantly deal with particular situations of legal coherence, such as a conflict of laws, as those solved by Private International Law analysis. This paper briefly presents our version of Intuitionistic Description Logic, called iALC for Intuitionistic ALC (ALC being the canonical classical description logic system). We also discuss the jurisprudence foundation of our system, and show how we can perform a coherence analysis of “Conflict of Laws in Space” by means of our system iALC. This

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paper reports work-in-progress on using this alternative definition of logical negation for building and testing legal ontologies and reasoning in AI.

2. JURISPRUDENCE AND INTUITIONISM

One of the main problems from jurisprudence (legal theory) is to make precise the use of the term “law”. In fact, the problem of individuation, namely, what counts as the unit of law, seems to be one of the fundamental open questions in jurisprudence. Any approach to law classification requires firstly answering the question “What is to count as one complete law?”1. There are two main approaches to this question. One is to take all (existing) legally valid statements as a whole. This totality is called “the law”. The coherence of “the law” plays a central role in this approach, whilst whether coherence is built-in by the restrictions induced by Nature in an evolutionary way, or whether it should be object of knowledge management, seems to be a long and classical debate. The other approach to law definition is to take into account all legally valid statements as being individual laws. This view, in essence, is harder to share with jurisprudence principles, since those are concerned with justifying the law. Both approaches agree with legal philosophy and jurisprudence stemming from the Legal Positivism tradition initiated by Hans Kelsen2. The latter, however, seems to be more suitable to Legal AI. It is also considered by theoreticians, at least partially, whenever they start considering ontological commitments, such as, taking some legal relations as primitive ones3, primary and secondary rule4 or even a two-level logic to deal with different aspects of law (see logic-of-imperation/logic-of-obligation from Bentham5). In fact, some Knowledge Engineering (KE) groups pursue this approach as a basis for defining legal ontologies. We also follow this route.

It is important to note that we avoid the use of any deontic logic, since this leads to well-known contrary-to-duty paradoxes and their variants. We

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also believe that “laws” are socially motivated mechanisms – according to legal positivism – and should be not considered in terms of truth. The work of Valente\(^6\) discusses why deontic logic does not properly distinguish between the normative status of a situation and the normative status of a norm (rule).

Our approach will take legal statements as individuals of a legal universe, instead of propositions. The latter will be used to classify the legal statements. The natural (socially established) precedence between individual legal statements will be associated to the pre-order relationship that characterizes the Kripke Semantics for Intuitionistic Logic IL. In this article we use the short form VLS to denote a valid legal statement.

In order to compare our approach to the classical one, we can use the simple task of negating a legal statement. Consider the proposition “Peter is liable”, and assume that Peter is under the legal age in Brazil. The VLS representing this situation, in classical logic, is obtained by negating the proposition. “Peter is not liable” is the VLS inhabiting the world, which belongs to the Brazilian collection of VLS. Classical logic, validating the tertium non datur principle (the excluded third principle says that \(\alpha \lor \neg \alpha\) holds always), classifies the negation of “Peter is liable” as “Peter is not liable”. Taking intuitionistic logic IL, one may have neither “Peter is liable” nor “Peter is not liable” as part of the Brazilian collection of VLS. The semantics of IL avoids classification by negating a proposition. Thus, using IL, it is possible to consistently have a legal situation where Peter is taken as liable, even if he is under the legal age, for example. For the classical case, not being liable is the same that being not liable.

From the semantic point of view, iALC seems to be well suited to model the Legal theoretic approach pursued by KE as described above. Let us consider an iALC model having as individuals each of the valid and possible legal statements. The \(\preceq\) relation is the natural hierarchy existing between these individual legal statements. For example, sometimes conflicts between legal statements are solved by inspecting the age of the laws (how old is the first edition in the legal system), the wideness enforcement scope of each law, etc. Any of these relations considered is an order relation. For example, “Theodor is vicariously liable by John” is legally dominated by “John is a worker of Theodor”, or “John and Theodor have an employment contract”. Any legal statement involving the civil liability of someone must precede

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any legal statement asserting that he/she is of legal age. If $C$ is a concept symbol in a description logic language, its semantics is the collection of legal statements representing a kind of legal situation. The main role of Intuitionism in our setting is the change of meaning it provides to negation of concepts, as well as to subsumption. As a matter of clarification, we discuss briefly another case involving negation of concepts regarding the usual approach, based on classic ALC to the ontological formalization of “the law" that includes the development of one or more domain ontologies to be used in validating the legal statements. Consider the statement, PeterIsLiable, and that Peter is under legal age. As it is usual, people under the legal age are not able to sign contracts. Consider a part of an (hypothetical) ontology of Private Ownership Law with concepts RentingContract, for the collection of valid statements on renting contracts, ∃hasTenant.RentingContract, for the collection of statements describing that someone is a legal tenant.

Thus, ∃hasLandlord.RentingContract is the collection of legal landlords. Of course, the concept PeterIsLiable should not belong to ∃hasTenant.RentingContract nor ∃hasLandlord.RentingContract. The use of intuitionistic ALC would not force us to consider the concept PeterIsNotLiable as inhabiting the world of individual legal statements. Nevertheless, using classical ALC would force us to consider PeterIsNotLiable, the negation of PeterIsLiable, as inhabiting one of ∃hasTenant.RentingContract or ∃hasLandlord.RentingContract.

The intuitionistic approach depicted in this paper basically consists of a model which includes all the possible valid legal statements and the natural precedence relation among them. Legal connections will be also considered when relating legal statements belonging to distinct “Law Systems”.

3. INTUITIONISTIC DESCRIPTION LOGIC iALC

Description logics are quite popular right now. However, mostly they are classically biased, in the sense that the negation ($\neg$) of a concept is simply its set-theoretical complement, with respect to the universe of individuals. Subsumption of concepts is set-theoretical inclusion. This seems to be enough for most of the usual applications. However, as discussed in the paper by de Paiva\(^7\), constructive description logics also make sense, both from

a theoretical and from a practical viewpoint. There are many ways of defining constructive description logics. In particular Mendler and Scheele have worked out an interesting system\textsuperscript{8}. They cite auditing of business as their preferred application. Aiming to provide a formal basis for legal AI, we follow a different path and describe a constructive version of ALC, based on the framework for constructive modal logics developed by Simpson in his PhD thesis\textsuperscript{9}. This framework was firstly developed by Brauner and de Paiva\textsuperscript{10} for Hybrid Logics.

\( \text{ALC} \) is a basic description language. Its concept formers are described by the following grammar:

\[
C, D ::= A \mid \bot \mid \top \mid \neg C \mid C \land D \mid C \lor D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C
\]

where \( A \) stands for an atomic concept and \( R \) for an atomic role. This syntax is more general than standard ALC in that it includes subsumption \( \sqsubseteq \) as a concept-forming operator. Negation can be represented via subsumption, \( \neg C = C \sqsubseteq \bot \), but we find it convenient to keep it in the language. The constant \( \top \) can also be omitted since it can be represented by \( \neg \bot \).

Following Mendler and Scheele we say a constructive interpretation of \( \text{iALC} \) is a structure \( \mathcal{I} = (\Delta^\mathcal{I}, \preccurlyeq^\mathcal{I}, \cdot^\mathcal{I}) \) consisting of a non-empty set \( \Delta^\mathcal{I} \) of entities in which each entity represents a partially defined individual; a refinement preordering \( \preccurlyeq^\mathcal{I} \) on \( \Delta^\mathcal{I} \), i.e., a reflexive and transitive relation; and an interpretation function \( \cdot^\mathcal{I} \) mapping each role name \( R \) to a binary relation \( R^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I} \) and each atomic concept \( A \) to a set \( A^\mathcal{I} \subseteq \Delta^\mathcal{I} \) which is closed under refinement, i.e., \( x \in A^\mathcal{I} \) and \( x \preccurlyeq^\mathcal{I} y \) implies \( y \in A^\mathcal{I} \).

The interpretation \( \mathcal{I} \) is lifted from atomic \( \bot, A \) to arbitrary concepts via:

\[
\begin{align*}
\top^\mathcal{I} &= \Delta^\mathcal{I} \\
(\neg C)^\mathcal{I} &= \{ x | \forall y \in \Delta^\mathcal{I} . x \preccurlyeq^\mathcal{I} y \Rightarrow y \notin C^\mathcal{I} \} \\
(C \land D)^\mathcal{I} &= C^\mathcal{I} \cap D^\mathcal{I} \\
(C \lor D)^\mathcal{I} &= C^\mathcal{I} \cup D^\mathcal{I} \\
(C \sqsubseteq D)^\mathcal{I} &= \{ x | \forall y \in \Delta^\mathcal{I} . (x \preccurlyeq^\mathcal{I} y \text{ and } y \in D^\mathcal{I}) \Rightarrow y \in C^\mathcal{I} \} \\
(\exists R.C)^\mathcal{I} &= \{ x | \forall y \in \Delta^\mathcal{I} . x \preccurlyeq^\mathcal{I} y \Rightarrow \exists z \in \Delta^\mathcal{I} . (y, z) \in R^\mathcal{I} \text{ and } z \in C^\mathcal{I} \} \\
(\forall R.C)^\mathcal{I} &= \{ x | \forall y \in \Delta^\mathcal{I} . x \preccurlyeq^\mathcal{I} y \Rightarrow \forall z \in \Delta^\mathcal{I} . (y, z) \in R^\mathcal{I} \Rightarrow z \in C^\mathcal{I} \}
\end{align*}
\]


Our setting is a simplification of Mendler and Scheele’s where we dispense with infallible entities, since our system iALC satisfies (like classical ALC) both $\exists R. \bot = \bot$ and $\exists R. (C \sqcup D) = \exists R. C \sqcup \exists R. D$. We will have no use for nested subsumptions, but they do make the system easier to define, so we keep the general rules.

Simpson’s original system is a constructive Natural Deduction system whose rules capture exactly the intuitions of the modalities over possible worlds. We simply adapt it to the description logic fragment. The rules are in Fig. 1. The labels intuitively describe the world where the concept is being asserted. Thus $x : C$ means that the VLS $x$ is in $C$. (In terms of Kripke models, $|x| = C$, where $x$ is a world). Our deductive system also has assertions of the form $xRy$, meaning that the role $R$ relates VLSs $x$ and $y$.

$$
\begin{array}{c}
\frac{\Gamma, x : C \Rightarrow x : C, \Delta}{\Gamma, x : C \Rightarrow \Delta, xRy} & \frac{xRy, \Gamma \Rightarrow \Delta, xRy}{\Gamma \Rightarrow x : C_1, C_1 \Rightarrow C_2} \\
\frac{C_1 \Rightarrow C_2}{x : C_1 \Rightarrow \Delta} & \frac{C_1 \Rightarrow x : C_2}{x : C_1 \Rightarrow \Delta} \\
\frac{\Gamma \Rightarrow C \Gamma, y, D \Rightarrow \Delta}{\Gamma, y, C \sqcup D \Rightarrow \Delta} & \frac{\exists R \Rightarrow D}{x : C \Rightarrow C \sqcup D} \\
\frac{\exists R \Rightarrow C \sqcup D}{\Gamma, y, C \sqcup D \Rightarrow \Delta} & \frac{\exists R \Rightarrow \Delta}{\Gamma, x \Rightarrow y : C} \\
\frac{\Gamma \Rightarrow x : (C \sqcup D), \Delta}{\Gamma \Rightarrow x : C \sqcup D, \Delta} & \frac{\exists R \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, x \Rightarrow 3R.C} \\
\frac{\exists R \Rightarrow \Delta, y \Rightarrow 3R.C}{\Gamma \Rightarrow \Delta, x \Rightarrow 3R.C} & \\
\frac{\exists R \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, x \Rightarrow 3R.C} \\
\end{array}
$$

Fig. 1 – iALC Rules

4. Private International Law Using iALC

A concept symbol $C$, in a description logic language, is associated to the subset of VLSs representing a kind of legal situation. Roles in the description logic language are associated to relations between these legal situations,
imposed by the relationship between each pair of individual legal statements. In order to prove that a VLS belongs to a legal situation, one may consider some of the VLSs that it precedes also in the concept. This is represented by the framed sequent in the proof below. Consider the following situation:

Peter and Maria signed a renting contract. The contract concerns an apartment in Rio de Janeiro. It states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20. Peter lives in Edinburgh and Maria lives in Rio.

Write \( \text{BR} \) for the collection of legal statements valid in Brazil, \( \text{SC} \) for the ones valid in Scotland, and \( c \text{mp} \) for the syntactic counterpart of Peter and Maria have contractual obligations to each other regarding... we will prove that the VLS \( c \text{mp} \) is in \( \text{BR} \), \( c \text{mp} : \text{BR} \). As 18 is the legal age in Brazil, there is no VLS stating Peter’s liability in Brazil. The statement Maria is liable, which we write as ml, is in \( \text{BR} \). Similarly, pl for Peter is liable is in \( \text{SC} \). Only liable people have civil obligations. Thus, \( c \text{mp} \leq pl \) and \( c \text{mp} \leq ml \). If we write PIL for Private International Law statements that are in \( \text{BR} \), then PIL \( \subseteq \text{BR} \). PIL is a disjunction of concepts of legal statements including \( \exists \text{LexDom.ABROAD} \). PIL relates legal statements in different contexts, locations, times, etc. Each member of PIL is concerned with a context, in particular ABROAD, AB in the proof, is the union of the VLSs holding in all countries, except Brazil. Lex-Domicilium is a legal connection, a relation between laws, LexDom in the proof. The pair \( \langle pl, pl \rangle \) is in LexDom, for Peter lives in Scotland, abroad as far as Brazil is concerned. Using \( \Delta \), the concept assumptions, and \( \Omega \), the subsumption assumptions, below we will prove that \( c \text{mp} : \text{BR} \).

Whenever a juridic case is described, its juridic context has to be taken into account. This context can be defined as the collection of legal statements that are legally valid and must precede the legal statement associated to the sentence of the considered case. In our case, the sentence is about the validity of the contract. The context that we consider is ml : BR, pl : BR \( \Rightarrow c \text{mp} : \text{BR} \). Considering contexts is strongly related to a Closed World Assumption (C.W.A.). Of course, during a trial, new legal statements can be considered. This has to do with the more complicated concept of “Juridical Process”, that will be not discussed in this article. The collection \( \Omega \) of subsumptions to be considered includes this C.W.A., just discussed.

We have the following set of axioms.

\[
\Delta = \{ml : \text{BR}, pl : \text{SC}, c \text{mp} \leq pl, c \text{mp} \leq ml, pl \text{LexDom} pl\}
\]

\[
\Omega = \{\text{PIL} \Rightarrow \text{BR}, \text{SC} \Rightarrow \text{ABROAD}, \ldots, \exists \text{LexDom.ABROAD} \subseteq \cdots \Rightarrow \text{PIL}
ml : \text{BR}, pl : \text{BR} \Rightarrow c \text{mp} : \text{BR}\}
\]
where $\Omega$ says $\text{PIL}_{\text{BR}} \Rightarrow \text{BR}$ states that (Brazilian) Private International Law holds in Brazil, $\text{SC} \Rightarrow \text{ABROAD}$ that Scottish law is among holds abroad Brasil and $\exists \text{LexDom}_{\text{ABROAD}}$ is among the alternative laws in the Brazilian Private International Law.

In the proof presented in Fig. 2 the sequents in $\Omega$ are used freely. The names of the concepts were shortened due to space limitations when displaying the proof tree. Since the complete proof would not fit in the page, we split it into two parts. We call $\Pi$ the first derivation and use it in the second derivation.

$$
\Delta \Rightarrow p:\text{BR} \\
\Delta \Rightarrow p:\text{AB} \quad \text{L-R} \\
\Delta \Rightarrow p:\text{LexDom}_{\text{AB}} \quad \exists \text{R} \\
\Delta \Rightarrow p:\text{LexDom}_{\text{AB}} \Rightarrow \text{PIL} \\
\Delta \Rightarrow \text{LexDom}_{\text{AB}} \Rightarrow \text{BR} \quad \text{inc-R} \\
\Delta \Rightarrow p:\text{BR}
$$

$$
\Pi \\
\Delta \Rightarrow m:\text{BR} \\
\Delta \Rightarrow p:\text{BR}, p:\text{BR} \Rightarrow \text{cmp} : \text{BR} \quad \text{cut} \\
\Delta \Rightarrow \text{cmp} : \text{BR} \\
\Delta \Rightarrow \text{cmp} : \text{BR} \quad \text{cut}
$$

Fig. 2 – Proof that $\text{cmp} : \text{BR}$

If one uses ALC instead of $i$ALC in the formalization, one will need to consider a legal ontology involving non-valid Legal Statements, and hence an ad hoc ontology regarding jurisprudence main concepts. Dealing with non-valid legal statements will increase the complexity of the ontology considered so far. Of course we simplified our example, since only $\text{PeterIsLiable}$ and $\text{MariaIsLiable}$ as succeeding contract is considered.

In a real ontology, many more statements would have to be considered, $\text{MariaOwnsTheApartment}$ is among them. Considering an actual juridic case, i.e., the legal statements a judge must take into account, would turn the formalization in ALC more complicated than its $i$ALC counterpart.

5. Conclusions

In this article we used $i$ALC, a constructive description logic, to provide an alternative, adequate, definition for subsumption, that copes with the jurisprudence theory that views “The Law” as all (possible) legally valid individuals laws. An example of conflict of laws, namely geographic conflict
of laws, was formalized by means of iALC in order to show its adequacy to perform coherence analysis in legal AI. Our previous work\textsuperscript{11} discusses the sequent calculus used here as well as the design of an automated reasoner for iALC. This sequent calculus is inspired by the work on the labeled sequent calculus for ALC presented in Rademaker et al.\textsuperscript{12}.

We indicated how the use of intuitionistic negation, instead of its classical counterpart, is adequate for Legal Reasoning. We consider it to be better than the classical approach, mainly when taking “legal statements” as elements of a domain to be formalized, and, not as propositions to be verified, as it is the case with deontic approaches to “Law” formalization.
