Vague Legal Concepts and Fuzzy Logic: An Attempt to Determine the Required Period of Waiting After Traffic Accidents

LOTHAR PHILIPPS

1.1. According to German law, a person involved in a traffic accident may not leave the site of the accident but is required to wait until somebody arrives who is willing to take his name and information on the car and the accident; otherwise he commits «unerlaubtes Entfernen vom Unfallort» (leaving the scene of an accident; § 142 I StGB). Of course nobody has to wait forever, but only for an «amount of time adequate under the circumstances».

The statute, however, does not tell us what amount of time is adequate under what circumstances. That information is nowhere to be found. Fritjof Haft tells us¹: «A rule of thumb might be: ‘The more severe the accident, the longer the required wait’. More cannot be said. Even a lawyer cannot tell you more, not even for money».

That is indeed very little. Comparative rules (the more severe – the longer) carry only little information. The rule does not say how bad an accident has to be to require a certain period of waiting.

Perhaps the rule of thumb given by Haft could be supplemented by another rule: «A severe accident requires a long wait; a medium accident requires a medium wait; and a slight accident only requires a short wait. Very extensive damage, esp. if people are killed or severely injured, necessitates a very long wait². If the damage is merely a scratch, one can leave immediately». This new rule joins an indeterminate time of required waiting with an equally open issue.

It does not give much more information, however. First, one has to take into account that terms like ‘slight’, ‘severe’, ‘short’, or ‘long’ are

² In order to prevent misunderstanding about the German law, the following should be noted: If e.g. a human life is at stake, one is not only allowed to leave the site of the accident to fetch help, but might even be required to do so; in that case the leaving is justified, but one has to report a near-by police station immediately afterwards. If the injured person is taken care of in another way, one has to remain at the site of the accident.
dependent on the context (compare the duration of a 'long faculty meeting' to that of a 'short term of imprisonment'). In addition, there are questions like: «What is still 'only a scratch', and what is 'light damage'? When does a 'medium wait' turn into a 'long wait'?»

In fact, one must be careful not to presuppose too much at this point. It should not be assumed the light damage stops at a certain point, at which medium damage starts. Or that a medium wait ends exactly when a long wait begins. Rather the terms run into one another, without clear boundaries; a fact long known to legal methodology³.

1.2. Traditional logic of course stipulates that an object either is or is not a member of a class – either totally or not at all. There is no room for gradual transitions. But for a quarter of a century fuzzy logic has existed, and has recently become a subject of general public interest⁴, triggered by a technological breakthrough the Japanese have achieved using it. They have managed to create appliances which adapt to their environment and live up to the unspoken expectations of their users. The advertisements promise «intelligent» cameras and «sensitive vacuum-cleaners».

In fuzzy logic the either-or classification does not exist. It is based on the idea that an item may also be part of a class to a greater or lesser extent.

This concept is also found in every-day speech. Somebody might not be «old» yet, but already «a little old». If two classes are neighboring concepts, it will often happen that an item extends into both. One is not necessarily either «young», «middle-aged» or «old», but perhaps «still pretty young, but already middle-aged». And somebody who is «still middle-aged» could already be called «old», if with less justification.

It is possible to use precise numerical expressions instead of vague qualities. The age could be given in years instead of terms of «young» and «old». But really, that is no solution to the problem. It will be possible to

³ To name a classic in this vast field: Grünhut, Begriffsbildung und Rechtsanwendung im Strafrecht (1926).
establish many precise delineators, but they will be too rigid. There is much which only applies to «young» people, and much that only applies to «old» people; but there is nothing that applies only to twenty-five year-olds, and not to twenty-six year-olds or twenty-four year-olds\textsuperscript{5}. In everyday speech numerical information is therefore often given with some vagueness, explicitly or implicitly: the number is a guide-line, not a delineator («thirtyish», «the woman of thirty years»).

Statements with vague terms are not only more flexible, but also more «durable». They can retain validity for a long time, even if their concrete significance varies with time. If indeed there is still a «dangerous age» for women, it would be much higher today than it was in Balzac’s day.

2. Philipp Heck has found a metaphor for vague terms which has had great influence in publications on legal theory: «The meaning [of a word] can be compared to a moon which is surrounded by a halo». «A nucleus of certain meaning is surrounded by a gradually fading halo of meaning\textsuperscript{6}». That is a good metaphor. I would like to add another metaphor, even more expressive and precise: a mountain skyline before the evening sky. The meaning of a word has a peak, from which it slopes down into the valley. It may be a narrow summit or a high plateau. The slopes into the valley may be steep or gentle, jagged or straight. When two mountains lie adjacent, their skylines often run into each other. The closer the mountains are to each other, the more they melt together.

The metaphor of a mountain skyline is more expressive and precise than that of nucleus and halo, because it translates directly to the graphs for the membership function of classes, and because a point in a graph is easier to define than a nuance of a fading hue. Also the relation of neighboring terms can be expressed more clearly.

3. The skyline for the required waiting period after a car accident might look like this\textsuperscript{7}:

\textsuperscript{5} With the exception of artificial prescriptions of the law-maker, such as coming of age at 18. Still a bright seventeen year-old takes better care of himself than a slow-witted eighteen year-old. The clear cut is accepted for the sake of general predictability of law.


\textsuperscript{7} A «very long» wait, as it should occur if deaths or severe injuries are involved, has been omitted, since it is only rarely necessary, as P. Gerathewohl demonstrates: Erschließung unbestimmter Rechtsbegriffe mit Hilfe des Computers. Ein Versuch am Beispiel der
The meaning of the expression «short wait» retains its peak up to twenty minutes; then it dips and finally ends at 40 minutes. Meanwhile the «medium wait» has started – also at 20 minutes. It peaks between 40 and 80 minutes and ends at 100 minutes, deeply within the area of the «long wait».

Thus, a remark by Adolf Merkels becomes directly evident: «Very many terms in jurisprudence have a flowing quality... Their areas of application are not separated by insurmountable fences, but rather there are steady transitions into the area of neighboring terms».

I have designed these graphs intuitively. It is a first suggestion which must be discussed by the specialists. But most of all, there have to be tests

«angemessen Wartezeit» bei 142 StGB (Doctoral thesis Tübingen 1987). In our thoroughly organized society the police will arrive with or shortly after the ambulance. And: One who decides to stay and wait after such an accident – and does not elope – will also wait a long time.

8 Quoted according to Grünhut, p. 16.

9 One could also present typical cases to experts for decision. The graph would express a sort of vote among judges, for example. If all of them subsumed an object (or an event) under a certain term, the object would receive the degree of membership of 1; if half of them subsumed the degree would be 0.5. Cf. Zadeh, Fuzzy Sets, Usurality etc. p. 5 («the grade of membership... as a measure or consensus»). Theoretically such vote could also be taken from published court rulings. I doubt, however, that there enough typical rulings.
to see whether such an understanding of the required waiting period will yield results which satisfy the sense of justice.

Especially the trapezoid or triangle will have to be viewed as a simplification and stylization. In reality, a term’s use will not be so rigid and linear. Rather, one would expect a smoothly curved meaning. But how does the curve run? As long as that is unknown, it is sensible to connect with straight lines the fixed points of one hypothesis – beginning, end and peak. It also has little impact whether an object belongs to a class at 0.4 or at 0.5 – and belongs to the neighbouring class to a certain degree. It does make a difference if an object totally belongs to one class and not at all to its neighbour.

There is a hierarchy of categories in these diagrams: at the top there are the «linguistic variables». In our case it is a term like «duration of the required wait».

The values of the linguistic variables – one step lower in the hierarchy – are not numerical, but of natural speech: terms such as «short wait», «medium wait».

Below these are the «base variables», in our case «minutes of waiting», whose values are finally numerical.

The numerical values are related to the values of natural speech, not in a definite way, but in the sense of a possible use in natural speech. Thus, there is an area where both are conceivable: a waiting period might be called «short» or «medium». This relation is therefore called a possibility distribution.

4. The graphs of the membership functions are very illustrative – comparable to the Euler Circles that are commonly used to symbolize relations between terms.

**Figure 2.**

The graphs are more expressive than simply circles. While the circles only signify whether one term is contained in the other wholly, partially, or not at all, the curves will also signify to what extent terms overlap or how far they lie apart. The graphs also express the semantic shape of a
term: for example whether its meaning gradually wanes or whether it breaks off abruptly.

I would like to demonstrate this using the graph for the linguistic variable "time of accident". Where the time of car accidents is concerned I have experimentally followed a dissertation on the "appropriate period of waiting", which distinguishes between "Day", "Evening", and "Night". The morning is missing. It might be mere coincidence that Gerathewohl, the dissertation's author, simply did not come across a ruling on an accident in the morning. Let us assume, however, that there are specific evening accidents, but no specific morning accidents: then the morning will have to be covered by the terms "Day" and "Night". Consequently the transition from "Night" to "Day" will be much more gradual than that between "Day" and "Evening" or between "Evening" and "Night". This is expressed in the diagram: the membership function graphs slope gently for the first transition, but steeply for the second and third (this demonstration has played part in my decision to use Gerathewohl’s terminology, despite my skepticism of his distinction).

**Figure 3. Time of Accident**

5.1. In designing a skyline for the variable "amount of damage" we encounter a difficulty which is probably inherent in most legal terms: it may be a huge difference whether someone caused damage of 50 DM or of 1,000 DM. It is, however, a small difference whether the damage comes
to 20,000 or 21,000 DM. On a small scale small differences matter, but on a larger scale only great differences matter. The base values have no linear relation to the linguistic variables. A simple solution might be a logarithmic scale for the membership function. The area of light damage would fall off slowly at first, and then ever more quickly. It might, for example, start at 50 DM and still have a degree of 0.8 at 1,300 DM, but have disappeared at 5,000 DM. Here the area of severe damage begins, which only peaks at 500,000 DM but already attains a degree of 0.8 at 20,000 DM. Medium damage would lie between the two, starting at 200 DM, peaking at 5,000 DM, and ending at 125,000 DM. Such a possibility distribution has proven sound after some experimentation.

**Figure 4. Damage**

Of course this distribution is only a suggestion, others should be tried. Here it is interesting to note how slowly changes take effect in fuzzy logic. This is because every term which is partially given is cushioned by a neighbouring term which is also partially given. My suggestion might be problematic in that the medium damage already commences at 200 DM. This, however, is cushioned by the fact that the light damage is still almost fully given at that point. Likewise at the other end, where medium damage carries up to 125,000 DM: that point lies practically in the area of full severe damage.
5.2. How do we deal with legal terms if their substratum – other than time or money – cannot be measured cardinally? Or if their substratum can theoretically be measured but not in the situation in question? It is a natural assumption that the duration of a required waiting period after a car accident is also dependent on the amount of traffic. This can easily be measured; but when there is an accident, it will be rare that there is somebody there counting.

I suggest using a simple, nominal scale, which might contain the levels «small», «medium», and «large» (or the corresponding variants such as «light», «medium», and «severe»). Numerical values are assigned to each of these expressions, creating a scale from 0 to 10. The scale is divided in a manner typical for fuzzy logic: «small» starts falling off at 2 and ends at 5. At 2 «medium» begins and peaks at 5, ending at 8. And so on. Next to the core of the term there are three points of diminished degree (the third signifying the end of the term).

**Figure 5. Traffic Density**

6. How do we express such vague terms in a legal rule\textsuperscript{10}? On the level of linguistic variables a rule schema might be worded like this:

\textsuperscript{10} F. Haft has expressed the opinion in the context of required waiting after car accidents that rules with general terms act as crutches for the human mind – as compensation for the
If a certain damage or injury has occurred, a certain period of waiting is required.

After the variables have been substituted in the structure, the resulting rules could be:
1) If light damage occurs, a short period of waiting is required.
2) If medium damage occurs, a medium period of waiting is required.
Finally, there is the level of base values, where damage is expressed in DM and waiting period in minutes.
The same process is followed for such terms as «place of accident» and «traffic density».
Such rules are used like this: let us assume that somebody has caused 1,300 DM worth of damage. In an attempt to apply our rules, we discover that the definitions of both «light damage» and «medium damage» are fulfilled – but neither fully: the first is satisfied to a degree of 0.80, the other to a degree of 0.55. Likewise, both conclusions «short wait required» and «medium wait required» are justified: again, the first is justified to a degree of 0.80, the other to a degree of 0.55.

We have several rules with overlapping conditions and consequences. A case satisfies several overlapping rules at the same time, whereby one is satisfied more, the other less. This yields several competing legal outcomes at the same time.
This, of course, is not a definite result. We need a method of determining a definite point of time up to which waiting is required, and after which leaving is not subject to punishment.
The first part of this process – the description of the factual situation using «vague» terminology – is called ‘fuzzification’. The second part – translating vague terms back into a decision – is called ‘defuzzification’. These terms were developed in control theory. They can, however, be easily applied to jurisprudence.
The process of fuzzification has already been described. There are several approaches to defuzzification. The most commonly employed one is the fact that it is not able to take into account a high number of relevant factors or to understand their connection. Computergerstüte Expertensysteme in der juristischen Aus- und Fortbildungs, in: Rechtsinformatik in den achtziger Jahren (1984), p. 20 ff. The computer, however, does not require this crutch. Does it really? Especially the latest fuzzy control technology places vague rules-of-thumb between the aim of a machine and the technical process. The rules-of-thumb express how a person would react in the machine’s position. If it is too cold turn the heat up to medium temperature. If it is only slightly too cold, only turn the heat up a little. If it is much too cold, turn it up all the way. Incidentally, I am perceiving some anthropomorphization of technology which deserves the attention of all the arts.
«Centroid Method». In our example, the rules we use result in three overlapping trapezoids representing the short, medium and long waiting periods. Let's assume all three trapezoids are present, but each only to a certain degree. The potential skyline, expressing the possible legal consequences, is partially filled. The part that is filled can be called the current skyline. The Centroid Method determines the gravitational center of the current skyline. Imagine cutting the skyline from heavy paper and then balancing it on your fingertip, finding the gravitational center. That point is used as the result.

This approach is evidently sound, since it literally yields well-balanced results. Extensive tests have shown that the Centroid Method gives the most satisfactory output\(^\text{11}\).

Defuzzification means a loss of information: skylines of totally different shapes may have the same gravitational center. But this loss of information is factually correct. It is obvious that radically different accidents might still require the same waiting period. On the other hand, it is obvious that one cannot deduct the circumstances of the accident from the required wait.

7. The different normative rules-of-thumb, which are utilized all at once to varying degrees, involve compromises. If we assume that the application of a rule yields the result that the short wait corresponds to a degree of 0.5, and the medium wait to a degree of 0.5, then the person involved in the accident has to wait an amount of time exactly half way between a short and a medium wait.

If we now add a second rule, with the result that the person only has to wait a short time, the gravitational center, and thus the decision, will move toward the short wait.

Competing legal rules usually do not behave this way: one prevails over the others. The exception replaces the norm, and the more specific rule replaces the more general rule.

However, there are legal norms open to compromise: These are usually called «principles» to distinguish them from legal rules\(^\text{12}\). The word principle carries a connotation of depth that I would not like to claim for normative rules-of-thumb. But why should only fundamental norms behave like principles?

The system I am employing, CubiCalc\(^\text{13}\), allows rules to be weighed

\(^\text{11}\) There are modifications of this method and varying other methods, which will not be discussed here.


\(^\text{13}\) Developed by Hyperlogic, Escondido, CA.
differently. The compromise between rules will then be tilted in favor of the more heavily weighted rule.

Weights can also be attributed in a variable manner, meaning that the weight of one rule is altered if another rule is activated. A borderline case would be a rule that reduces another's weight to zero if activated; it would completely replace the other. For example, if the accident happens on a holiday, one might consider reducing the weight of the rule for the time of day. On holidays there is no rush-hour.

8.1. Let us now regard the system’s architecture. The duration of the required wait is determined mostly by two factors: the amount of damage to the other party, and the level of expectation that someone will arrive at the site to record the facts of the accident. That the duration of the wait is dependent on the amount of damage has already been said; it is evident. Regarding the goal of ascertaining the facts: the more reason there is to expect someone will arrive to record the facts, the less cause there is to leave the site of the accident. And vice versa: the more unlikely it is that anyone will arrive, the less reason there is to wait, and the less justification there is to require the person to remain at the site.

The expectation of a passer-by arriving is not something that could be recognized directly, but there are various factors dermining the level of expectation. Following Gerathewohl, I have used place and time of the accident and the amount of traffic. Gerathewohl also considers whether the accident took place on a weekday or holiday, and whether the accident was clearly visible. I have disregarded these criteria for now since they play only a minor part in Gerathewohl’s statistics. Also I have weighted all rules equally.

8.2. Apart from the two factors of damage and expectation of somebody’s arrival, there is a third factor involved in determining the wait, a factor which acts as a catch-all for those circumstances that indicate a shorter than normal waiting period is required. A long period cannot reasonably be expected if the weather is very bad, or the person involved in the accident is ill, or has an important appointment, e.g. at court. The required

---

14 This relation is not as evident as that between damage and required waiting period. Some of partners in discussion – jurists and others – have been of the opinion that the required period should not be directly but inversely proportional to the expectation of someone's arrival. The smaller the probability that anyone arrives, the longer the wait should be (should be in a normative, not factual-prognostic sense – perhaps this double meaning is the basis of the disagreement). The lawyer of course has to decide at this point; no logic in the world can take this burden of his shoulders. Fuzzy logic can only help where we cannot clearly say what we want – not where we do not know what we want.
wait can also be reduced if the legal situation is obvious and the person makes it clear in some fashion that he will accept responsibility.

I have – so far – not included this third factor into the system. There are two reasons: important appointments are usually characterized by a certain time of day (say 9:00 am) which cannot be expressed in terms of duration (e.g. 15 minutes). And if the system were to decide whether there were extenuating circumstances, I think there would be problems with its acceptance. People might accept or even welcome an objective system which determines the required waiting period; but they will object to a machine which evaluates the degree of their personal hardship – e.g., a wait in freezing winter weather. The basic required waiting period determined by the system might therefore have to be modified by hand.

9. I will now add two diagrams which exemplify the system’s behavior. The graphs should be discussed from the viewpoint of whether the decision represented is proper and fair. If not, the decision graphs can easily be altered, mainly by adjusting the membership functions and, if that does not help, by changing the normative rules-of-thumb. Important court rulings should be taken into account when drawing the graphs\(^{15}\).

\(^{15}\)On the other hand it will rarely be possible to induce harmonious graphs from such rulings. If one considers for example the rulings listed by Grathewohl (p. 152 f), it becomes evident that for the first three or four criteria (the most important ones according to Grathewohl) certain patterns are repeated; but the corresponding waiting period varies wildly. The basis for induction is not necessarily contradictory, but at least very weak.
The diagrams show that the required waiting period depended on the time of the accident. The five graphs express different amounts of damage: 50-500-5,000-50,000-500,000 DM. In the first diagram the expectation that somebody will arrive to ascertain the facts is minimal (night, out in the country, no traffic); in the second diagram the expectation is maximal (day, center of a city, heavy traffic).

If there is little reason to expect someone to arrive, only a short wait is required after small damage, and also with more severe damage it does not reach the maximum waiting period. It is interesting to note that the slight difference between 50 DM and 500 DM is clearly noticeable, but the difference between 50,000 DM and 500,000 is hardly so.

Given the maximum expectation of someone’s arrival, the results are opposite: the maximum wait is required for very extensive damages (500,000 DM). The still considerable damage of 50,000 DM carries less weight. But even slight damage requires considerable waiting, while differences between small values (between 50 and 500 DM) are negligible. Such differences in the graphs should be discussed from a legal point of view (please note how the intermediate term «evening» takes effect on the right, while on the left «morning» is missing).

**Figure 7. Waiting period when someone is unlikely to arrive**
*(out in the country, no traffic)*
Figure 8. Waiting period when someone is very likely to arrive
(center of the city, heavy traffic)

Digression

1. A group of lawyers including Hans-Joachim Koch, Adalbert Podlech, and Helmut Rüßmann, best represented by the collection of essays «Juristische Methodenlehre und analytische Philosophie» (1976)¹⁶, do not suppose to be continuously many ways for a term to apply to an object rather than there are only three possibilities. They distinguish between a term’s «positive examples», where it clearly applies, a term’s «negative examples», where it clearly does not, and a term’s «neutral examples», whose relation to the term is indeterminate. The idea that a neutral candidate could tend more or less toward the positive or the negative side, is not accepted¹⁷.

If we use this concept to draw a membership graph for the «candidates», it will resemble a battlement of neighbouring terms without slopes:

The horizontal sections of the graph are on three possible levels: posi-

¹⁶ Ed. H.-J. Koch.
¹⁷ This idea was first offered by S. Körner, Experience and Theory (1966), and in the legal field by Podlech A. Podlech, Wertungen und Werte im Recht, in: «AÖR», Vol 95 (1970), p. 185 ff.
tive (1), neutral (.5), and negative (0). Their length can vary, of course, depending on whether the area of positive, negative or neutral candidates of a term is more or less extensive.

The regular and angular quality of the skyline seems to suggest that we are dealing with a very rough representation of reality.

2. Ulfried Neumann has argued against this trichotomy\(^{18}\), saying that it would be an illusion to believe that there are clear borders between the neutral examples and the positive or negative examples.

From the point of view of fuzzy logic the solution would be this: if I doubt whether I am dealing with a positive or a neutral example, but I am sure it is not a negative example, I will not use a membership value of exactly 0.5 (as I would for a clearly neutral example), but I would probably use a degree of 0.75 or 0.9 depending on how strong my doubts still are.

The problem mentioned by Neumann thus loses most of its relevance, but remains in theory. As soon as the subsumption of an object under a certain term is unclear, the exact graph for the membership function expressing this uncertainty can also be unclear. This is especially true for those border-line points at which, in Heck’s image, the bright core of the term begins to fade into the halo, or at the point where the halo disappears into the black of the surrounding night. Using the image of the mountain skyline, it is the point at which the high plateau starts sloping down into the valley, and again the point at which the valley is reached. If the uncertainty curve is in itself uncertain, the term «ultrafuzziness» is applied\(^{19}\).

I interpret Koch and Rüßmann’s argument in this way: they consider indeterminate legal terms to be simply fuzzy and use an extremely simple membership function, while Neumann considers them ultrafuzzy. I am also of the opinion that there are legal problems in which terms appear ultrafuzzy. For my current purposes it is not necessary to deal with the complexity of ultrafuzziness. I want to solve the practical problem of developing a system which reacts to differences in the facts of cases in a consistent and harmonious way. I am considering every-day cases, not border-line cases. The human mind is already lost in trying to cope with the many possible permutations of every-day cases, each of which poses no problem in itself. In this respect, and only in this respect, should the machine assist man.

---

\(^{18}\) Rechtsontologie und juristische Argumentation (1979), p. 71 ff.

\(^{19}\) Cf. Zadeh, Fuzzy Sets, Usuality etc, p. 6.